

Indian Statistical Institute
Mid-Semestral Examination
Topology III - MMathII

Max Marks: 40

Time: 120 minutes.

Give proper justification(s) for your answers.

- (1) Define the notion of a CW -complex and a subcomplex of a CW -complex. Show that a subcomplex is also a CW -complex. Does the subspace $X = \{1/n\} \cup \{0\}$ of \mathbb{R} have the homotopy type of a CW -complex? Justify. [3+4+3]
- (2) Let X be a CW -complex and X^p its p -skeleton. Show that the inclusion map $j : X^p \hookrightarrow X$ induces an isomorphism
- $$j_* : H_i(X^p) \longrightarrow H_i(X)$$
- for all $i \leq p$. Show that j_* need not be an isomorphism in degree $p + 1$. [8+2]
- (3) Using results proved in the class compute the homology groups of $\mathbb{R}P^n, \mathbb{R}P^\infty$. Also compute their cohomology groups with \mathbb{Z}, \mathbb{Z}_2 and \mathbb{Q} coefficients. [10]
- (4) Show that $\mathbb{R}P^2$ is not a retract of $\mathbb{R}P^3$ [5]
- (5) Present an argument that shows that the homology groups of a finite CW -complex are finitely generated abelian groups. Give an example of a simply connected CW -complex whose homology groups are not finitely generated. [2+3]